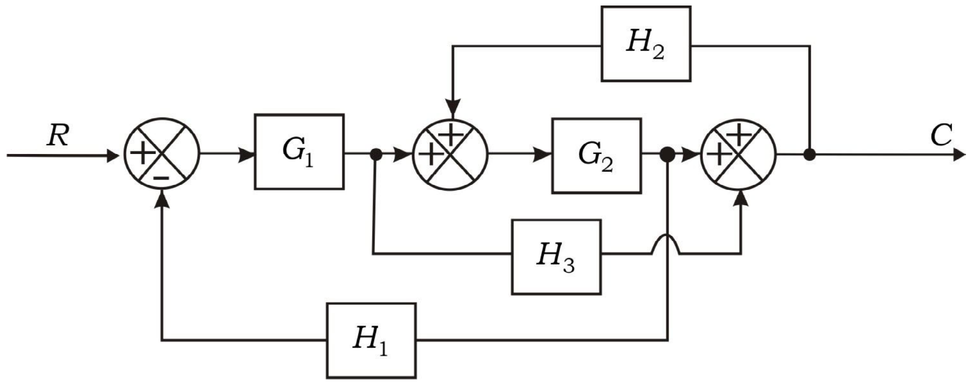
Q.1 For the system shown the equivelent T.F. is



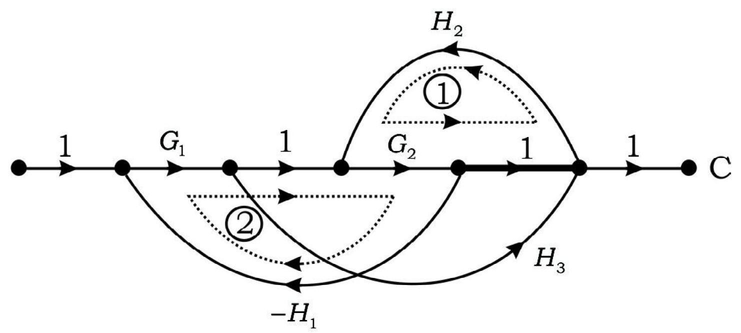
(a) $G_1\left[\dfrac{G_2+H_3}{1+G_1G_2H_1-G_2H_2}\right]$ (b) $\dfrac{G_1G_2+G_2H_3}{1+G_2H_1-G_1G_2H_2}$

(c) $\dfrac{G_2H_3}{1+G_2H_1+G_1G_2H_2}$ (d) $\dfrac{G_1G_2H_3}{1-G_1H_1-G_2H_2H_3}$

Ans. (a)

Exp.

The signal flow graph is



Number of forward paths n = 2

Gain along 1^{st} forward path

$$M_1 = 1.G_1.1.G_2.1.1 = G_1G_2$$

$$M_2 = 1.G_1.H_3.1 = G_1H_3$$

Two touching loops

$$\Delta = 1-\{-G_1G_2H_1+G_2H_2\}$$

$$\Delta_1 = 1, \Delta_2 = 1$$

$$T.F. = \frac{M_1\Delta_1+M_2\Delta_2}{\Delta} = \frac{G_1G_2+G_1H_3}{1+G_1G_2H_1-G_2H_2}$$

Q.2 Consider a ufb system with forward path T.F. is $G(S)=\dfrac{K(s+2)(s+4)}{(s-1)(s-3)}$. The range of K for which system is unstable

(a) $K < \frac{2}{3}$ (b) $K > -1$

(c) $-1 < K < \frac{2}{3}$ (d) $-1 > K$ or $K > \frac{2}{3}$

Ans. (c)

Exp.

The characterstie equation $1 + G(S) = 0$

$(s-1)(s-3) + K(s+2)(s+4) = 0$

$(K+1)s^2 + (6K-4)s + (8K+3) = 0$

Routh Hurwitz table

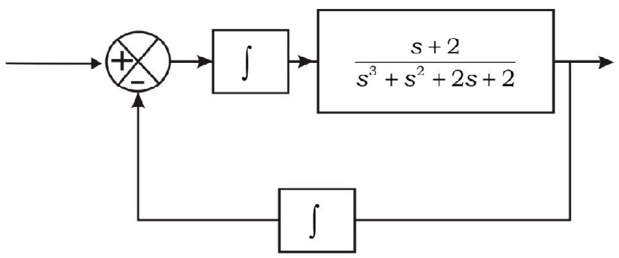
s^2	$K+1$	$8K+3$
s^1	$6K-4$	0
s^0	$8K+3$	0

For stable system there should not be any sign change in 1^{st} column either all -ve or all +ve

$$\left.\begin{array}{lll} K+1<0 & \Rightarrow & K<-1 \\ 6K-4<0 & \Rightarrow & K<2/3 \\ 8K+3<0 & \Rightarrow & K<-3/8 \end{array}\right\} \Rightarrow K<-1$$

$$\left.\begin{array}{lll} K+1>0 & \Rightarrow & K>-1 \\ 6K-4>0 & \Rightarrow & K>2/3 \\ 8K+3>0 & \Rightarrow & K>-3/8 \end{array}\right\} \Rightarrow K>2/3$$

Q.3 For the system shown



The number of poles in RH & on $j\omega$-axis are

(a) 0 & 0 (b) 2 & 0

(c) 3 & 2 (d) 3 & 0

Ans. (b)

Exp.

Integrator $\boxed{\int} = \frac{1}{s}$

$$G(s) = \frac{s+2}{s\left(s^3+s^2+2s+2\right)}$$

$$H(s) = \frac{1}{s}$$

Ch. equation $1 + G(s)H(s) = 0$

$$1 + \frac{s+2}{s^2\left(s^3 + s^2 + 2s + 2\right)} = 0$$

$$\Rightarrow s^5 + s^4 + 2s^3 + 2s^2 + s + 2 = 0$$

s^5	1	2	1
s^4	1	2	2
s^3	$0(\varepsilon)$	-1	0
s^2	$\frac{2\varepsilon+1}{\varepsilon}$	2	0
s^1	$-\frac{2\varepsilon^2+\varepsilon+1}{2\varepsilon+1}$	0	0
s^0	2	0	0

The 1^{st} column if $\varepsilon \to 0 \quad (\varepsilon \geq 0)$

1	$+ve$
1	$+ve$
ε	$+ve$
$2 + \frac{1}{\varepsilon}$	$+ve$
$-\frac{2\varepsilon^2+\varepsilon+1}{2\varepsilon+1}$	$-ve$
2	$+ve$

there ar two sign changes so 2-roots are in R.H. of s-plane.
Complete - zero row is not observed so no root on $j\omega -$ axis

Q.4 The O.L.T.F for a *ufb* system is $\frac{K}{s^2+2s-3}$. The value of K for which two closed loop poles are lying on R.H. side of line $s = -2$

(a) $K > 3$ (b) $K < 3$
(c) $K > 0$ (d) $K < 0$

Ans. (a)
Exp.

The Ch. equation

$$1 + \frac{K}{s^2 + 2s - 3} = 0$$

$$\Rightarrow \quad s^2 + 2s + (K - 3) = 0$$

Let $z = s + 2$

$\Rightarrow \quad s = z - 2$

$$(z-2)^2 + 2(z-2) + K - 3 = 0$$

$$\Rightarrow z^2 - 2z + (K-3) = 0$$

R.H. Table

z^2	1	$K-3$
z^1	-2	0
z^0	$K-3$	0

For two roots are lying on R.H. side of $s = -2$ i.e. for which

$\text{Re}[s] > -2 \Rightarrow \text{Re}[z] > 0$

Hence two roots are lying on R.H.S. of z-plane $z > 0$ so there should be two sign changes Hence

$K - 3 > 0 \quad \Rightarrow \quad K > 3$

Q.5 The OLTF of a ufb system is $G(s) = \dfrac{K}{s^m(s+\alpha)}$.

The system has a setting time 0.4 sec and position error constant 10. The value of K & α are

(a) 10 & 10 (b) 100 & 100

(c) 10 & 100 (d) 100 & 10

Ans. (d)

Exp. As position error constant $K_P = 10$ i.e. finite number Hence it is a type -0 system, i.e. $m = 0$

$$G(s) = \frac{K}{s+\alpha} = \frac{K/\alpha}{1 + s/\alpha}$$

Compare with $G(s) = \dfrac{K_P}{1+sT}$

$$K_P = \frac{K}{\alpha} = 10, \quad T = \frac{1}{\alpha}$$

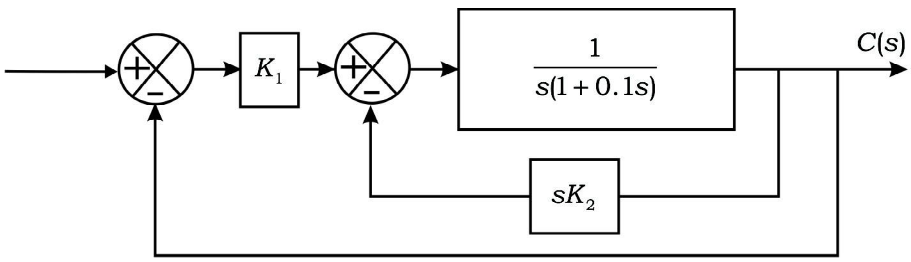
Settling time $t_s = 4T = \dfrac{4}{\alpha} = 0.4$

$\Rightarrow \quad \alpha = 10$

& $K = 10\alpha = 100$

Common Date for Q. 6 and Q. 7

For a control system shown



Q.6 If $K_2 = 0$, determine the value of K_1 for an steady state error of 0.1

(a) 10 (b) 20

(c) 5 (d) 6

Ans. (a)

Exp. If $K_2 = 0$

$$OLTF = \frac{K_1}{s(1+0.1s)}$$

Compare with $G(s) = \dfrac{K_v}{s(1+sT)}$

Velocity error constant $K_v = K_1$

S.S error for ramp input

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K_1} = 0.1$$

$\Rightarrow$ $K_1 = 10$

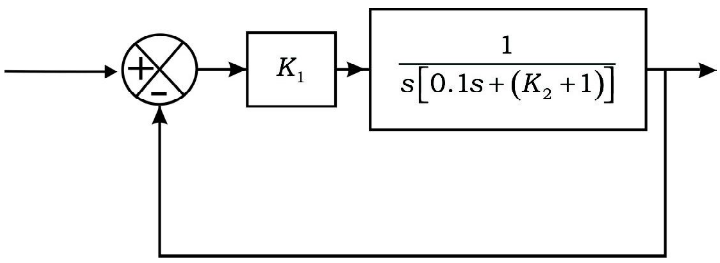
Q.7 If the damping ratio is to be changed to 0.8 without affecting S.S. error, the value of K_1 & K_2 are

(a) 32, 2.2 (b) 25.6 & 1.56

(c) 11.23, 1.12 (d) 10 & 0

Ans. (b)

Exp. Inner loop function = $\dfrac{\dfrac{1}{s(1+0.1s)}}{1+\dfrac{sK_2}{s(1+0.1s)}} = \dfrac{1}{s[K_2+1+0.1s]}$



$$OLTF = \frac{K_1}{s[(K_2+1)+0.1s]}$$

$$= \frac{K_1/(K_2+1)}{s\left[1+\dfrac{0.1}{K_2+1}s\right]}$$

Velocity error constant $K_v = \dfrac{K_1}{K_2+1}$

$$e_{ss} = \frac{1}{K_v} = \frac{K_2+1}{K_1} = 0-1$$

$$\Rightarrow K_2 + 1 = 0.1K_1$$

Characteristic eq^n

$$1 + \frac{K_1}{s\left[\left(K_2+1\right)+0.1s\right]} = 0$$

$$\Rightarrow 0.1s^2 + s\left(K_2+1\right) + K_1 = 0$$

$$s^2 + 10\left(K_2+1\right)s + 10K_1 = 0$$

Compare with $s^2 + 2\rho\omega_n s + \omega_n^2 = 0$

$$\omega_n = \sqrt{10K_1},\ 2\rho\omega_n = 10\left(K_2+1\right)$$

$$\Rightarrow \quad \rho = \frac{5\left(K_2+1\right)}{\sqrt{10K_1}} = 0.8$$

$$K_2 + 1 = 0.1K_1$$

$$\frac{5\times 0.1\,K_1}{\sqrt{10K_1}} = 0.8$$

$$\Rightarrow \quad K_1 = 25.6$$

$$K_2 = 0.1K_1 - 1 = 1.56$$

Q.8 The unit impulse open-loop system response of a system is $g(t) = 1 - e^{-t}$, $t \geq 0$. Now if unity feedback is provided, the steady state errors for unit ramp and unit parabolic input are

(a) 1, 0 (b) ∞, ∞

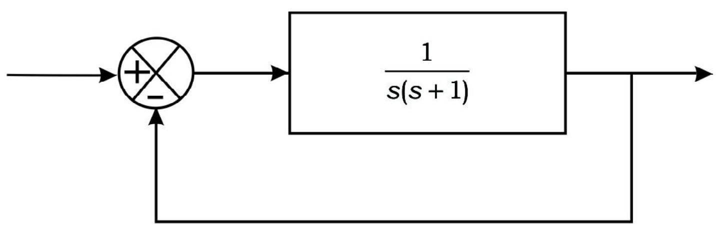
(c) $\infty, 1$ (d) $1, \infty$

Ans. (d)

Exp. $g(t) = 1 - e^{-t}$

$$O.L.T.F.\quad G(s) = \pounds\left[g(t)\right] = \frac{1}{s} - \frac{1}{s+1}$$

$$= \frac{1}{s\left[1+s\right]}$$



It is type -1 system with

velocity error constant $K_v = 1, \quad K_a = 0$

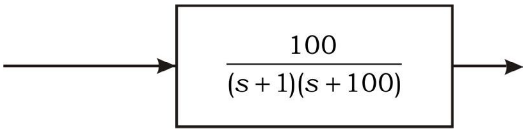
Hence S.S error for unit ramp input

$$e_{ss} = \frac{1}{K_v} = 1$$

For unit parabalic input

$$e_{ss} = \frac{1}{K_a} = \infty$$

Q.9



For the system shown above. The time required by the response to reach upto 99% of its steady state value is

(a) 46 ms (b) 400 sec

(c) 4.6 sec (d) 4 sec

Ans. (c)

Exp.

$$G(s) = \frac{100}{(s+1)(s+100)}$$

$$= \frac{1}{(1+s.1)\left(1+s.\frac{1}{100}\right)}$$

Campare with $= \dfrac{1}{(1+sT_1)(1+sT_2)}$

Time constants

$$T_1 = 1, \; T_2 = \frac{1}{100}$$

As $\quad T_1 >> T_2$

The response is dominated by higher time constant

$$G(s) \simeq \frac{1}{1+s}$$

Time cosnstant $T = 1\text{sec}$.

For unit step input (say) $R(s) = 1/s$

$$C(s) = G(s)\, R(s) = \frac{1}{s(s+1)}$$

$$= \frac{1}{s} - \frac{1}{s+1}$$

$$c(t) = 1 - e^{-t}$$

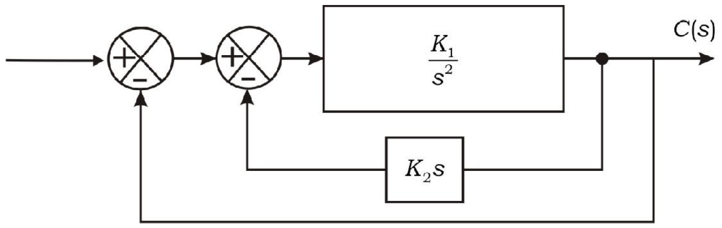
S.S $\; c_{ss} = 1$

At $\quad t = t_x, \; c = 0.99 \times c_{ss} = 0.99$

$\Rightarrow \quad 0.99 = 1 - e^{-t_x}$

$\Rightarrow \quad t_x = 4.6\,\text{sec}.$

Q.10



If the damping factor and settling time of the system are 0.8 & 5 rad/sec respectively. The values of K_1 & K_2 are respectively

(a) 5, 0.16
(b) 25, 0.32
(c) 10, 2
(d) 50, 2.5

Ans. (b)

Exp.

Inner loop T.F. $G(s) = \dfrac{K_1 / s^2}{1 + K_2 s.\dfrac{K_2}{s^2}}$

$$= \frac{K_1}{s^2 + K_1 K_2 s}$$

$$G(s) = \frac{K_1}{s\left(s + K_1 K_2\right)}$$

Char. $eq^n \quad 1 + G(s) = 0$

$$1 + \frac{K_1}{s\left(s + K_1 K_2\right)} = 0$$

$\Rightarrow \quad s^2 + K_1 K_2 s + K_1 = 0$

Compare with $s^2 + 2\rho\omega_n s + \omega_n^2 = 0$

$$\omega_n^2 = K_1, \quad 2\rho\omega_n = K_1 K_2$$

$$\Rightarrow \omega_n = \sqrt{K_1}, \quad \rho = \frac{K_1 K_2}{2\omega_n} = \frac{K_1 K_2}{2\sqrt{K_1}}$$

$$\rho = \frac{K_2}{2}\sqrt{K_1}$$

$$\omega_n = \sqrt{14}$$

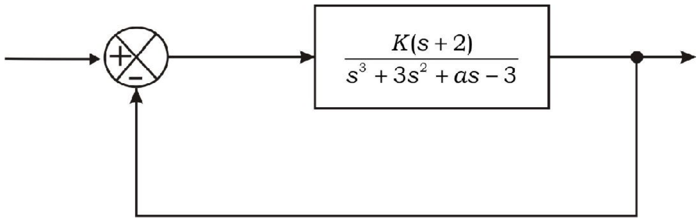
$$\Rightarrow K_1 = \omega_n{}^2 = 25$$

$$\rho = \frac{K_2}{2}\sqrt{K_1}$$

$$0.8 = \frac{K_2}{2} \times 5$$

$\Rightarrow \; K_2 = 0.32$

Common Data for Q.11 & 12



Where $K > 0$ & $a > 0$

Q.11 The range of K for which the system is stable

(a) $K < \frac{3}{2}$ (b) $K > \frac{3}{2}$

(c) $K > 0$ (d) $K > 3$

Ans. (b)

Q.12 The values of K & a so that the sustained oscillation occur

(a) $3, \frac{5}{2}$ (b) $\frac{3}{2}, 2$

(c) $\frac{3}{2}, 5$ (d) Not possible

Ans. (d)

Exp. Ch. equation

$$1 + \frac{K(s+2)}{s^3+3s^2+as-3} = 0$$

$$\Rightarrow s^3 + 3s^2 + (K+a)s + (2K-3) = 0$$

s^3	1	$K+a$
s^2	3	$2K-3$
s^1	$\frac{K+3a+3}{3}$	0
s^0	$2K-3$	0

For stability

$2K - 3 > 0 \;\Rightarrow K > 3/2$

For sustained oscillations i.e. marginally stable system (Complete row zero)

$$\frac{K+3a+3}{3} = 0$$

But Both K, a > 0

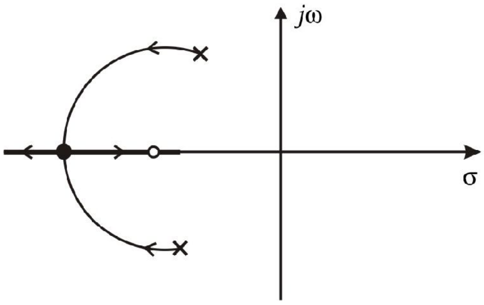
$$\Rightarrow \quad \frac{K+3a+3}{3} \neq 0$$

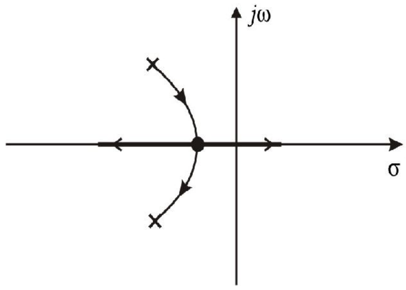
Hence marginal stable not possible.

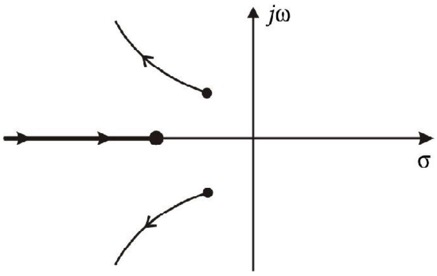
Common Data for Q.13, 14 & 15

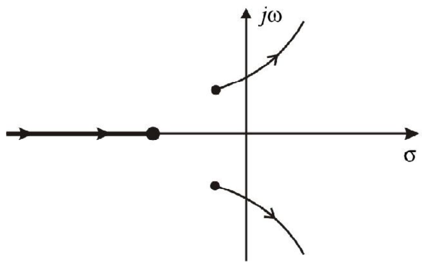
The characteristic equation of a control system is given as $1+\frac{K(s+2)}{s^2+2s+2}=0$

Q.13 The root locus plot

(a) 

(b) 

(c) 

(d) 

Ans. (a)

Q.14 The system gives critically damped response if

(a) $0<K<4.83$ (b) $K=4.83$

(c) $K>11.26$ (d) $K=11.26$

Ans. (b)

Q.15 The system gives under damped and overdamped response if

(a) $K<4.83$ & $K>4.83$ (b) $K<11.26$ & $K>11.26$

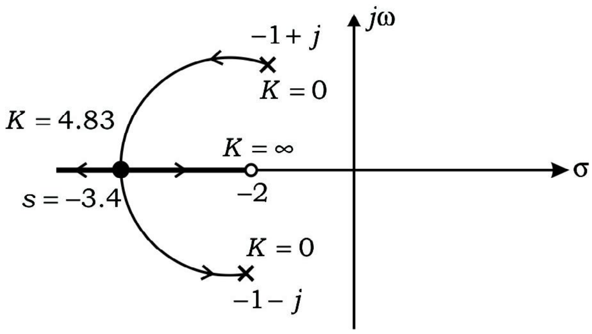
(c) $K=4.83$ & $K\neq 4.83$ (d) $K\neq 4.83$ & $K=4.83$

Ans. (a)

Exp. $$OLTF=\frac{K(s+2)}{s^2+2s+2}$$

OL Zeros $m=1,\quad s=-2$

OZ pols $n=2,\quad s=-1\pm j$



Num. of asymptotes $= n - m = 1$

Angle of Asymptote $\varphi_A = 180^o$

Break away point

$$K = -\frac{s^2 + 2s + 2}{s + 2}$$

$$\frac{dK}{ds} = -\frac{(s+2)(2s+2) - (s^2 + 2s + 2).1}{(s+2)^2} = 0$$

$$\Rightarrow\ s^2 + 4s + 2 = 0$$

$$\Rightarrow\ s = -0.6,\ -3.4$$

$s = -0.6$ does not lie an root locus

For $s = -3.4$ break away point

$$K = -\frac{(-3.4)^2 + 2\times(-3.4) + 2}{-3.4 + 2}$$

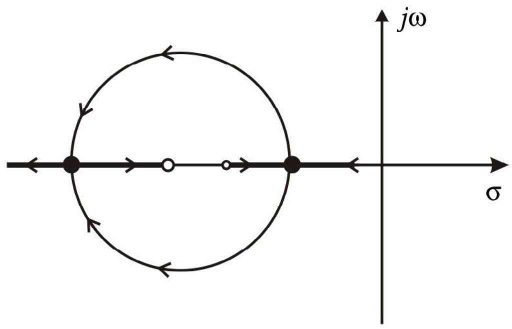
$$= 4.83$$

At $K - 4.83$ both roots are an -ve real axis, i.e. coincident means the system is critically damped

if $0 < K < 4.83$ Complex canjugate roots Hence the response is under damped

If $K > 4.83$ Both roots on -ve real axis and distinct home the response is over damped

Q.16 The root locus of the control system $1 + \frac{K(s+4)}{s(s+3)} = 0$ is given as



The centre & radius of the circle in root locus is given as

(a) (–5, 0) & 3
(b) (–6.5, 0) & 4.5
(c) (–4, 0) & 2
(d) (–8, 0) & 6

Ans. (c)

Exp. $$\frac{K(s+4)}{s(s+3)} = 1$$

$$s = \sigma + j\omega$$

$$\frac{K\left[(4+\sigma)+j\omega\right]}{(\sigma+j\omega)\left[(3+\sigma)+j\omega\right]}=-1=1\underline{|180^\circ}$$

$$\Rightarrow \quad \underline{\left|\frac{(\sigma+4)+j\omega}{(\sigma+j\omega)\left[(\sigma+3)+j\omega\right]}\right.}=180^\circ$$

$$\Rightarrow \quad \underline{|(\sigma+4)+j\omega}-\left[\underline{|\sigma+j\omega}+\underline{|(\sigma+3)+j\omega}\right]$$

$$=180^0$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{\omega}{\sigma+4}\right)-\tan^{-1}\left(\frac{\omega}{\sigma+3}\right)=180^0+\tan^{-1}\frac{\omega}{\sigma}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{\omega}{\sigma+4}-\frac{\omega}{\sigma+3}}{1+\frac{\omega^2}{(\sigma+3)(\sigma+4)}}\right]=180^0+\tan^{-1}\frac{\omega}{\sigma}$$

$$\frac{\omega(\sigma+3)-\omega(\sigma+4)}{(\sigma+3)(\sigma+4)+\omega^2}=\frac{\omega}{\sigma}$$

$$\Rightarrow -\sigma=\sigma^2+7\sigma+12+\omega^2$$

$$\Rightarrow \sigma^2+8\sigma+\omega^2+12=0$$

$$\Rightarrow (\sigma+4)^2+\omega^2=4$$

Centre of circle $(-4,0)$ radius = 2

Q.17 For a unity feedback system the OLTF $G(s)=\frac{1}{s(1+s)^2}$.

The gain margin of system is

(a) 0.5 (b) 2

(c) 1 (d) 0

Ans. (b)

Exp. Frequency response $s+j\omega$

$$G(j\omega)=\frac{1}{j\omega(1+j\omega)^2}$$

Gain

$$M=\left|G(j\omega)\right|=\frac{1}{\omega\left(1+\omega^2\right)}$$

Phase $\varphi=\underline{|G(j\omega)}=-90^\circ-2\tan^{-1}\omega$

At phase cross over freq. $\omega_\rho \quad \varphi=-180^\circ$

$$-180^\circ = -90^\circ - 2\tan^{-1}\omega_P$$

$\Rightarrow$ $2\tan^{-1}\omega_P = 90^\circ$

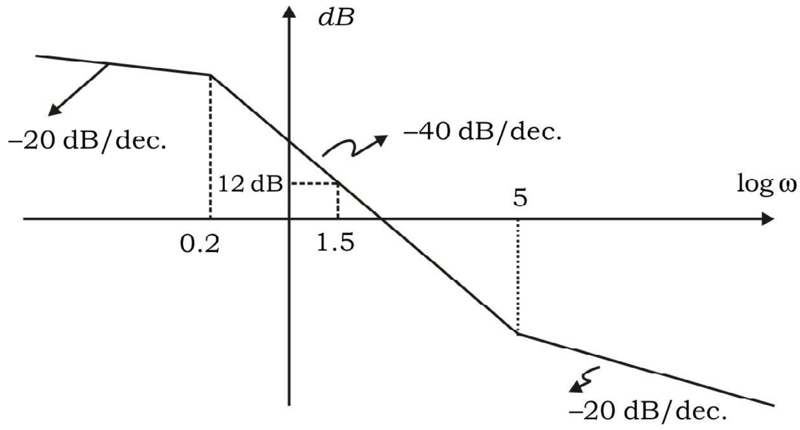
$\Rightarrow$ $\omega_P = 1$

Gain at $\omega_P = 1$

$$M = \frac{1}{1.\left(1+1^2\right)} = \frac{1}{2}$$

Gain margin $GM = \dfrac{1}{Gain\ at\ \omega_P} = \dfrac{1}{1/2} = 2$

Q.18 The gain plot for a minimum phase function is represented as



The T.F. of the system is (approximately)

(a) $\dfrac{3(s+5)}{s^2(5s+1)}$ (b) $\dfrac{15(5s+1)}{s(s+5)}$

(c) $\dfrac{9(s+5)}{s(5s+1)}$ (d) $\dfrac{12(s+5)}{s^2(5s+1)}$

Ans. (c)

Exp.

The corner frequencies are 0.2 rad./sec & 5 rad/sec.

Lowest corner freq. $\omega_{cl} = 0.2$ rod/sec

The initial slope, i.e. slope at frequencies less than the lowest corner freq.

$= -20\,dB\ /\ dec$

so its is Type - 1 system

so a term of $\dfrac{K}{s}$ exists

$$G(s) = \frac{K}{s}\frac{(1+s/5)}{(1+s/0.2)}$$

$$G(j\omega) = K\frac{\left(1+j\dfrac{\omega}{5}\right)}{j\omega\left[1+j\dfrac{\omega}{0.2}\right]}$$

At $\omega < 5$
Gain in dB

$$dB = 20\log\left|G(j\omega)\right| = 20\log K - 20\log\omega - 20\log\left[1+\left(\frac{\omega}{0.2}\right)^2\right]^{\frac{1}{2}}$$

$$dB = 20\log K - 20\log\omega - 20\log(5\omega)$$

$$= 20\log K - 40\log\omega - 20\log 5$$

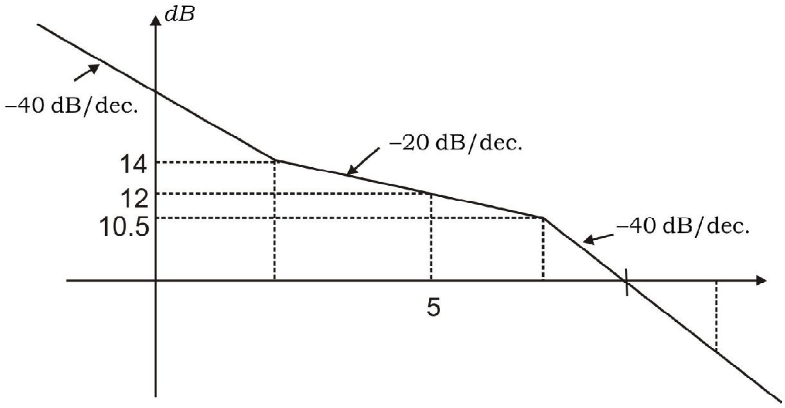
At $\omega = 1.5$, $dB = 12$

$$12 = 20\log K - 40\log 1.5 - 20\log 5$$

$$K = 44.8 \simeq 45$$

$$G(S) = \frac{45}{s}\frac{(1+s/5)}{(1+5s)} = \frac{9(s+5)}{s(5s+1)}$$

Q.19



The T.F. of minimum phase function

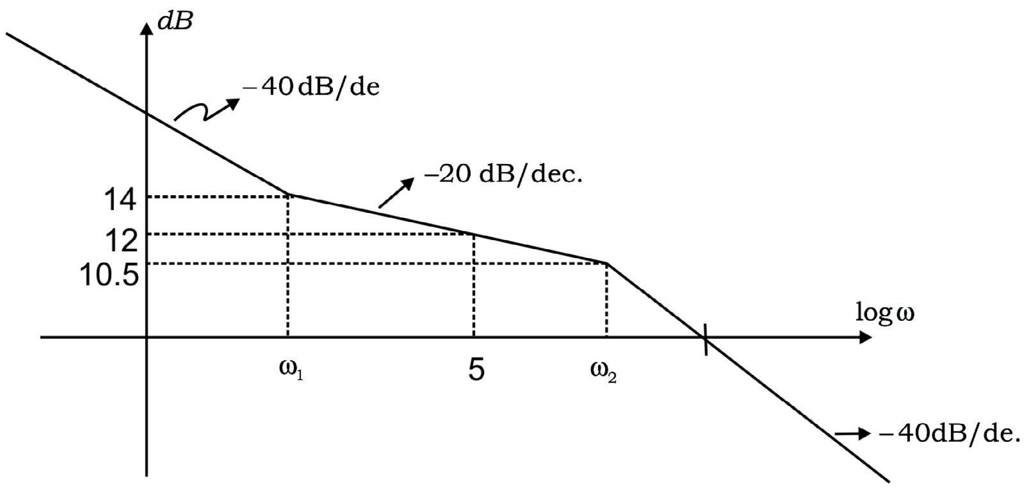
(a) $\frac{100(s+2)}{s^2(s+6)}$

(b) $\frac{120(s+2)}{s^2(s+3)}$

(c) $\frac{120(s+4)}{s^2(s+5)}$

(d) $\frac{120(s+4)}{s^2(s+6)}$

Ans. (d)
Exp.



The corner freq. are ω_1 & ω_2

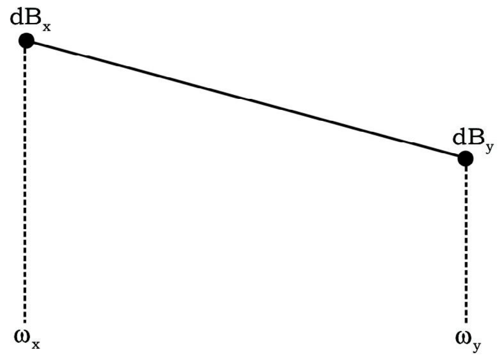
Intial slope $= -40\ dB\ /\ dec$

so it is Type-2 system

mean a term of $K\ /\ s^2$

$$G(S) = \frac{K}{s^2}\frac{\left(1+s\ /\ \omega_1\right)}{\left(1+s\ /\ \omega_2\right)}$$

If slope of line M



$$dB_x - dB_y = M\ \log_{10}\frac{\omega_x}{\omega_y}$$

For $\quad M = -20\ dB\ /\ dec.$

$$\omega_x = \omega_1,\quad dB_x = 14$$

$$\omega_y = 5,\quad dB_y = 12$$

$$14 - 12 - = 20\log\frac{\omega_1}{5}$$

$\Rightarrow \qquad \omega_1 \simeq 4\quad rad\ /\sec.$

$$G(s) = \frac{K\left(1+s\ /\ 4\right)}{s^2\left(1+s\ /\ \omega_2\right)}$$

At $\qquad \omega \le 4$

$$G(s) \simeq \frac{K}{s^2} = \frac{K}{\left(j\omega\right)^2}$$

$$dB = 20\log K - 40\log\omega$$

At $\qquad \omega = \omega_1 = 4,\quad dB = 14$

$$14 = 20\log K - 40\log 4$$

$$\Rightarrow K \simeq 80$$

$$G(s) = \frac{80\left(1+s\ /\ 4\right)}{s^2\left(1+s\ /\ \omega_2\right)}$$

At $\omega_x = 5, \; dB_x = 12$

$\omega_y = \omega_2 \quad dB_y = 10.5$

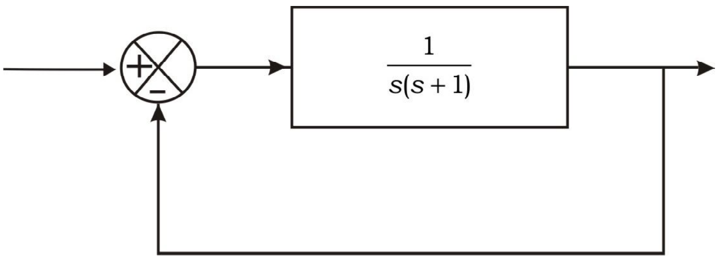
$$12 - 10.5 = -20 \log \frac{5}{\omega_2}$$

$\Rightarrow \quad \omega_2 \simeq 6$

$$G(S) = \frac{80(1+s/4)}{s^2(1+s/6)}$$

$$= \frac{120(s+4)}{s^2(s+6)}$$

Q.20 For the control system shown



The sensitivity of CLTF wrt OLTF at $\omega = 1$ rad/sec is

(a) $\sqrt{2}$ (b) 0.2

(c) $\frac{1}{\sqrt{2}}$ (d) 2

Ans. (a)

Exp. $OLTF \quad G = \frac{1}{s(s+1)}$

$$CLTF \quad T = \frac{G}{1+G}$$

$$S_G^T = \frac{\partial T / T}{\partial G / G} = \frac{G}{T}\frac{\partial T}{\partial G}$$

$$= \frac{1}{1+G} = \frac{1}{1+\frac{1}{s(s+1)}}$$

$$= \frac{s^2+s}{s^2+s+1}$$

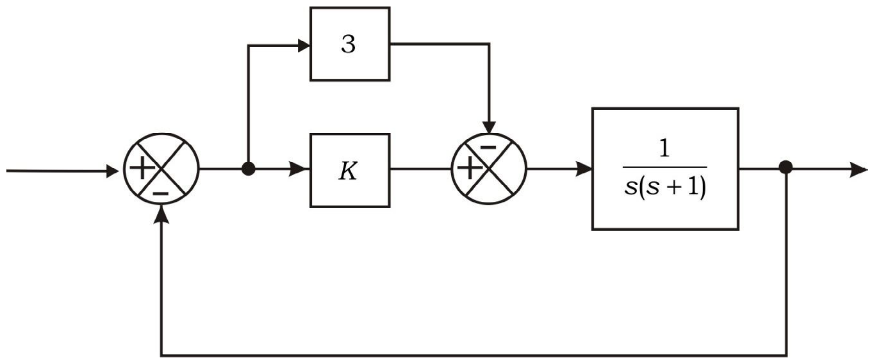
$s = j\omega$

$$S_G^T(j\omega) = \frac{j\omega - \omega^2}{1-\omega^2+j\omega}$$

At $\omega = 1$

$$S_G^T(j1) = \frac{-1+j}{j} = 1+j = \sqrt{2}\angle 45^\circ$$

Q.21



The maximum possible sensitivity of CLTF wrt OLTF is at $\omega = 1$

(a) 2

(b) $\sqrt{\frac{2}{17}}$

(c) 1

(d) $\sqrt{2}$

Ans. (d)

Exp.

$$OLTF \quad G(s) = \frac{K-3}{s(s+1)}$$

Ch.equation $\quad 1+G(s)=0$

$$s^2+s+K-3=0$$

For stable operation $K>3$

$$S_G^T = \frac{1}{1+G} = \frac{1}{1+\frac{K-3}{s(s+1)}}$$

$$= \frac{s^2+s}{s^2+s+K-3}$$

$$s = j1$$

$$S_G^T(j1) = \frac{-1+j}{-1+j+K-3} = \frac{-1+j}{(K-4)+j}$$

$$\left|S_G^T\right| = \sqrt{\frac{2}{(K-4)^2+1}}$$

For $\left|S_G^T\right|_{max}$ $\quad K-4=0$

$$\Rightarrow K = 4$$

$$\left|S_G^T\right| = \sqrt{2}$$

Common Data for Q.22 & 23
The state space representing matrix are given

$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $C = [1 \quad 0]$

In initial state $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Q.22 The state transition matrix

(a) $\begin{bmatrix} \sin t & \cos t \\ -\cos t & \sin t \end{bmatrix}$ (b) $\begin{bmatrix} \sin t & \cos t \\ \cos t & -\sin t \end{bmatrix}$

(c) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$ (d) $\begin{bmatrix} \cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix}$

Ans. (c)

Q.23 The response for unit step input is

(a) $\sin t + \cos t - 2$ (b) $2(1 - \cos t)$

(c) $2(1 + \cos t)$ (d) $\sin t + t$

Ans. (b)
Exp.

$$[sI - A] = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}$$

$$\varphi(s) = [sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}^{-1}$$

$$= \frac{1}{s.s. - (1)(-1)} \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{s^2+1} & \frac{1}{s^2+1} \\ -\frac{1}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix}$$

S.T.M

$$\varphi(t) = \pounds^{-1}[\varphi(s)]$$

$$= \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$$

$$X(s) = \varphi(s)x(0) + \varphi(s)B\,U(s)$$

$$= \frac{1}{s^2+1}\begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}\frac{1}{s}$$

$$= \frac{1}{s\left(s^2+1\right)}\begin{bmatrix} 2 \\ 2s \end{bmatrix} = \frac{2}{s\left(s^2+1\right)}\begin{bmatrix} 1 \\ s \end{bmatrix}$$

output $\quad Y(s) = C\,X(s)$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \dfrac{2}{s\left(s^2+1\right)} \\ \dfrac{2}{s^2+1} \end{bmatrix}$$

$$= \frac{2}{s\left(s^2+1\right)} = \frac{2}{s} - \frac{2s}{s^2+1}$$

$$Y(t) = 2 - 2\cos t$$

Q.24 For a system with the transfer function

$G(s) = \dfrac{2(s-3)}{s^3+2s^2-3s+1}$, the matrix A in the state space from $\dot{x} = Ax + Bu$ is equal to

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -3 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 6 & 3 \end{bmatrix}$

Ans. (a)
Exp.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s-6}{s^3+2s^2-3s+1}$$

$$Y(s) = -6.\frac{1}{s^3+2s^2-3s+1}U(s) + 2s.\frac{1}{s^3+2s^2-3s+1}U(s)$$

Let $\quad Y_1(s) = \dfrac{1}{s^3+2s^2-3s+1}U(s)$

$$s^3Y_1 + 2s^2Y_1 - 3sY_1 + Y_1 = U(s)$$

$\Rightarrow \quad \dddot{y} + 2\ddot{y}_1 - 3\dot{y}_1 + y_1 = u$

Let $\quad y_1 = x_1,\ \dot{y}_1 = x_2,\ \ddot{y}_1 = x_3$

$\Rightarrow \quad \dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 + 2x_3 - 3x_2 + x_1 = u$

$\Rightarrow \quad \dot{x}_3 = -x_1 + 3x_2 - 2x_3 + 4$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$Y(s) = -6Y_1 + 2sY_1$

$y = -6y_1 + 2\dot{y}_1$

$y = -6x_1 + 2x_2$

Q.25 A unity feedback control system has a forward loop T.F. as $\frac{e^{-s/2}}{1+s}$. The phase cross over frequency is given by

(a) $\omega = \cot\left(\frac{\omega}{2}\right)$

(b) $2\omega = \tan\omega$

(c) $2\omega + \cot\omega = 0$

(d) $\omega + \tan\left(\frac{\omega}{2}\right) = 0$

Ans. (d)

Exp. $G(s) = \frac{e^{-s/2}}{1+s}$

$$G(j\omega) = \frac{e^{-j\omega/2}}{1+j\omega}$$

phase $\quad \varphi = \angle G(j\omega) = -\frac{\omega}{2} - \tan^{-1}\omega = -180^\circ$

$$\tan^{-1}(\omega) = 180^\circ - \frac{\omega}{2}$$

$$\omega = \tan\left(180^\circ - \omega/2\right)$$

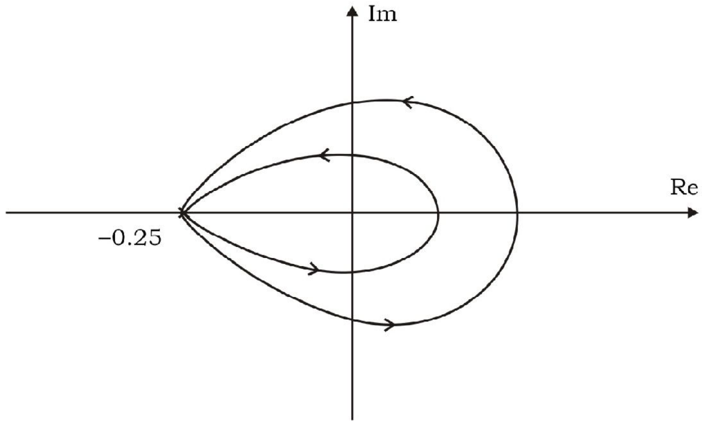
$\Rightarrow \quad \omega = -\tan(\omega/2)$

$\Rightarrow \quad \tan(\omega/2) + \omega = 0$

Q.26 The Nyquist plot of a unity feedback system having OLTF

$$G(s)=\frac{K(s+a_1)(s+a_2)}{(s-b_1)(s-b_2)}$$

where $a_1, a_2, b_1, b_2 > 0$ for $K=1$ is given by



For closed loop system to be stable the range of K is

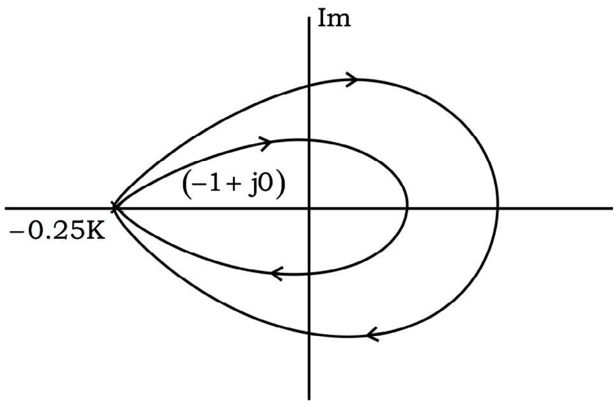
(a) $K < 4$ (b) $K > 4$

(c) $0 < K < 0.25$ (d) $0 < K < 2$

Ans. (b)

Exp.

For K



If $(-1+j0)$ point inside

$0.25K > 1 \quad \Rightarrow K > 4$

There are two encirclement in anti clockwise direction N = +2

Num. of OL poles in RH P = 2

Num of CL poles in RH = Z

$N = P - Z$

$+2 = 2 - Z$

$\Rightarrow \quad Z = 0$

Q.27 The matrix A of stable space representation of a control system is given as $A=\begin{bmatrix}0 & 1\\ -2 & a\end{bmatrix}$. The value of '$a$' for stability is

(a) $a > 2$ (b) $a < 2$
(c) $a < 0$ (d) $a > 0$

Ans. (c)
Exp.

$\Rightarrow$ Ch. equation

$$|sI - A| = 0$$

$$sI - A = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & a \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s-a \end{bmatrix}$$

$$|sI - A| = \begin{vmatrix} s & -1 \\ 2 & s-a \end{vmatrix} = s(s-a) + 2 = 0$$

Ch. equation

$$s^2 - as + 2 = 0$$

$$\begin{array}{c|cc} s^2 & 1 & 2 \\ s^1 & -a & 0 \\ s^0 & 2 & 0 \end{array}$$

For stability $-a > 0$

$\Rightarrow$ $a < 0$

Q.28 For a lead network having T.F.as $\frac{1+0.1s}{1+0.04s}$. The maximum phase shift is

(a) 38.6° (b) 25.4°
(c) 22.8° (d) 30°

Ans. (b)
Exp.

$$G_c(s) = \frac{1+0.1s}{1+0.04s}$$

Compare with $G_c(s) = \frac{1+s\tau}{1+s\propto\tau}$

$$\tau = 0.1 \qquad \alpha\tau = 0.04$$

$\Rightarrow$ $\alpha = 0.4$

Maximum phase shift

$$\tan\varphi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$= \frac{1-0.4}{2\sqrt{0.4}}$$

$\Rightarrow$ $\varphi_m = 25.4^\circ$

Q.29 For *ufb* OLTF

$$G(s) = \frac{2e^{-s}}{2s+1}$$

The phase margin is

(a) 70.15° (b) –109.85°

(c) 109.85° (d) 250.85°

Ans. (a)

Exp.

$$G(j\omega) = \frac{2e^{-j\omega}}{1 + j2\omega}$$

Gain

$$M = |G(j\omega)| = \frac{2}{\sqrt{1 + (2\omega)^2}}$$

At gain cross-over freq. ω_g

$$M = 1$$

$$\frac{2}{\sqrt{1 + (2\omega_g)^2}} = 1$$

$$\Rightarrow \quad 1 + (2\omega_g)^2 = 4$$

$$\Rightarrow \quad \omega_g = 0.87 \quad \text{rod/sec.}$$

$$\varphi = \underline{|G(j\omega)} = -3\omega - \tan^{-1}(2\omega)$$

At ω_g

$$\varphi = -\omega_g - \tan^{-1}(2\omega_g)$$

$$= -0.87 - \tan^{-1}(1.73)$$

$$= -109.85^o$$

$$GM = 180^o + \varphi = 70.15^\circ$$

Q.30 For a second order system the damping ratio is 0.7 the PM of system (approximately) is

(a) 30° (b) 40°

(c) 70° (d) 56°

Ans. (c)

Exp. PM (approx)

$$PM \simeq 100\rho \text{ (degress)}$$

$$PM \simeq 100 \times 0.7$$

$$\simeq 70^o$$